OPTIMIZING IN LIBRARY WORK

Miranda P. Vlad, Mircea V. Diudea

Abstract: *Optimizing the learning activity, by minimization of the walk length for the books inspection, as an important fact in education, can be achieved by using the Graph Theory. In this respect, three library models were designed as graphs. The distance inside the graphs was evaluated by the Wiener index (i.e. the sum of all distances in the graph). Centrality, counting the distribution of distances in the graph, was computed as the centrality index C (developped at TOPO GROUP Cluj). The walk length optimisation was proposed to be equivalent with the most centralized library structure, or the maximum C-index.*
Key-words*: learning optimization; graph, layer matrix, centrality index.*

Introduction

In the learning process quick access to information is very important.

While traditional libraries have lost much ground to the online ones, they still have an important role in learning and beyond, especially since many of the books are not available on the Internet.

In this regard we considered optimizing access to books from a classical library. The walk distance (and time) needed for inspection of the books lying at various levels/floors of a library has to be minimized if the optimal educational process is desired. In this paper we will use the Graph Theory as a theoretical background and TOPOCLUJ software package developped by TOPO Group Cluj, at the „Babes-Bolyai" University in Cluj.

Graph theoretical background

A *graph* $G(V,E)$ is a pair of two sets, V and E, $V=V(G)$ being a finite nonempty set and $E=E(G)$ a binary relation defined on V. (Harary 1969, Diudea 2010. A graph can be visualized by representing the elements of V by points (*i.e.*, vertices) and joining pairs of vertices (i,j) by a bond (*i.e.*, edge) if and only if $(i,j)\in E(G)$. The number of vertices in G equals the cardinality $v=|V(G)|$ of this set; analogously, the number of edges is given by the cardinality $e=|E(G)|$.

A *path* p_n of G is a subgraph whose vertices $v_1,v_2,.,v_n$ are all-distinct and no branching is allowed. The length of the path is $l(p_n)=|E(p_n)|$.

A graph is said connected if any two vertices *i* and *j* are the endpoints of a path; otherwise it is disconnected.

The *distance* d_{ij}, between two vertices *i* and *j*, is the length of a *shortest* path joining them, if any : $d_{ij} = min\ l(p_{ij})$; otherwise $d_{ij} = \infty$. A shortest path is often called a *geodesic*. The *set of all distances* (*i.e.*, geodesics) in *G* is denoted by *D*(*G*). (Diudea 2010, Diudea, Florescu and Khadikar 2006).

The *eccentricity* of a vertex *i* ecc_i in a connected graph is the maximum distance between *i* and any vertex *j* of *G*: $ecc_i = max\ d_{ij}$. The *radius* of a graph, *r*(*G*), is the minimum eccentricity among all vertices *i* in *G*: $r(G) = min\ ecc_i = min\ max\ d_{ij}$. Conversely, the *diameter* of a graph, *d*(*G*), is the maximum eccentricity in *G*: $d(G) = max\ ecc_i = max\ max\ d_{ij}$.

A square table, of dimensions $n \times n$, whose entries are defined as:

$$[\mathrm{A}(G)]_{ij} = \begin{cases} 1 & \text{if} \quad i \neq j \ \text{ and } \ (i,j) \in E(G) \\ 0 & \text{if} \quad i = j \ \text{ or } \ (i,j) \notin E(G) \end{cases} \qquad (1)$$

is called the *adjacency matrix* of *G*, **A**(*G*) and it characterizes the graph up to isomorphism. A(*G*) is symmetric vs. its main diagonal, so that the transpose $A^T(G)$ leaves A(*G*) unchanged.

Distance matrix **DI**(*G*) was introduced in 1969 by Harary (1969). It is a square symmetric table, of dimensions $n \times n$, whose entries are defined as: (Harary 1969; Diudea 2010, Diudea, Florescu and Khadikar 2006).

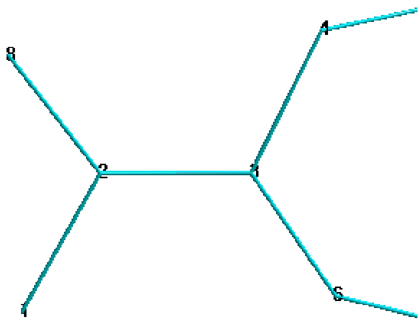
$$[\mathrm{DI}(G)]_{ij} = \begin{cases} \min l(p_{i,j}), \text{ if } \ i \neq j \\ 0 \text{ if } \ i=j \end{cases} \qquad (2)$$

The non-diagonal entries are just the topological distances between *i* and *j*.

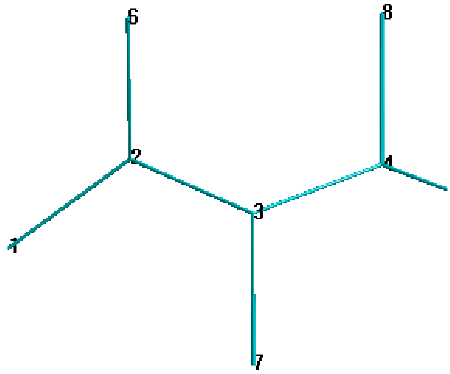
The half sum of all entries in DI(*G*) provides the well-known topological index Wiener *W*: (Wiener 1947; Diudea and Gutman 1998).

$$W = W(G) = (1/2)\sum_i \sum_j [\mathrm{DI}]_{ij} \qquad (3)$$

To exemplify the above notions, we consider two graphs, G_1 and G_2 on 8 vertices.



G_1



G_2

Figure 1 presents the adjacency and distance matrices of the above graphs.

Layer matrices (Diudea, Topan and Graovac 1994; Diudea 1994; Diudea and Ursu 2003) are built on layer partitions in a graph. Let $G(v)_k$ be the k^{th} layer of vertices v lying at distance k, in the partition $G(i)$:

$$G(v)_k = \{v \mid d_{i,v} = k\} \tag{4}$$

$$G(i) = \{\, G(v)_k \; ; k \in [0, 1, .., ecc_i] \,\} \tag{5}$$

with ecc_i being the *eccentricity* of i .

The entries in the layer matrix (of vertex property) **LM**, are defined as (Diudea and Ursu 2003).

$$\mathrm{LM}_{i,k} = \sum_{v | d_{i,v} = k} p_v \tag{6}$$

Layer matrix is a collection of the above defined entries:

$$\mathrm{LM}(G) = \{\, [\mathrm{LM}]_{i,k}; i \in V(G); k \in [0, 1, .., d(G)] \,\} \tag{7}$$

with $d(G)$ being the diameter of the graph (*i.e.*, the largest distance in G).

AD(G_1)

0	1	0	0	0	0	0	0	1
1	0	1	0	0	0	0	1	3
0	1	0	1	0	1	0	0	3
0	0	1	0	1	0	0	0	2
0	0	0	1	0	0	0	0	1
0	0	1	0	0	0	1	0	2
0	0	0	0	0	1	0	0	1
0	1	0	0	0	0	0	0	1
1	3	3	2	1	2	1	1	14

AD(G_2)

0	1	0	0	0	0	0	0	1
1	0	1	0	0	1	0	0	3
0	1	0	1	0	0	1	0	3
0	0	1	0	1	0	0	1	3
0	0	0	1	0	0	0	0	1
0	1	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	1
0	0	0	1	0	0	0	0	1
1	3	3	3	1	1	1	1	14

DI(G_1)

0	1	2	3	4	3	4	2	19
1	0	1	2	3	2	3	1	13
2	1	0	1	2	1	2	2	11
3	2	1	0	1	2	3	3	15
4	3	2	1	0	3	4	4	21
3	2	1	2	3	0	1	3	15
4	3	2	3	4	1	0	4	21
2	1	2	3	4	3	4	0	19
19	13	11	15	21	15	21	19	134

1/2SUM = 67 = W(G_1)

DI(G_2)

0	1	2	3	4	2	3	4	19
1	0	1	2	3	1	2	3	13
2	1	0	1	2	2	1	2	11
3	2	1	0	1	3	2	1	13
4	3	2	1	0	4	3	2	19
2	1	2	3	4	0	3	4	19
3	2	1	2	3	3	0	3	17
4	3	2	1	2	4	3	0	19
19	13	11	13	19	19	17	19	130

1/2SUM = 65 = W(G_2)

Figure 1. A and DI matrices for the graphs G_1 and G_2

On **LM** matrices, particularly **LC** (the Layer matrix of Counting) one calculates the following *index of centrality* C(LM): (Diudea 2010; Diudea and Ursu 2003; Vlad and Diudea 2013)

$$C(\mathrm{LM})_i = \left[\sum_{k=1}^{ecc_i} \left([\mathrm{LM}]_{ik}^{2k} \right)^{1/(ecc_i)^2} \right]^{-1} \tag{8}$$

$$C(\mathrm{LM}) = \sum_i C(\mathrm{LM})_i \tag{9}$$

This index allows the finding of the graph center and provides an ordering of graph vertices according to their centrality. A larger C_i-

value denotes a more central vertex and the largest value is found for the center of *G*; next, a larger C-value means a more centralized graph. Figure 2 illustrates the LC matrix of the two above graphs.

The half sum of the columns in the LC matrix represent the coeficients of Hosoya polynomial (counting the distance sequences in *G*); the first derivative, in x=1, equals the Wiener index (see above the distance matrix and its half sum).

In studies on centrality of graphs, Bonchev *et al*. (Bonchev, Mekenyan and Balaban 1989) have proposed the distance-based criteria *1D-3D* as follows:

1D*:* minimum vertex eccentricity; **min** ecc_i

2D*:* minimum vertex distance sum; **min** DIS_i

3D*:* minimum number of occurrence of the largest distance; **min** $[LC]_{i,j\,max}$

When applied hierarchically, the above criteria lead to the center(s) of a graph. The centrality idea can be seen in Figure 2, both as the 3D-criterion and C-values, for the two graphs considered.

LC(G_1)

1	1	2	2	2	1	0.2041
1	3	2	2	0	2	0.2752
1	3	4	0	0	**3**	**0.3904**
1	2	2	3	0	4	0.2639
1	1	1	2	3	5	0.1989
1	2	2	3	0	6	0.2639
1	1	1	2	3	7	0.1989
1	1	2	2	2	8	0.2041
8	14	16	16	10		**C = 1.9993**

$H(G_1) = 7x + 8x^2 + 8x^3 + 5x^4$

$H'(G_1) = 7+16+24+20 = 67 = W(G_1)$

LC(G_2)

1	1	2	2	2	1	0.2041
1	3	2	2	0	2	0.2752
1	3	4	0	0	**3**	**0.3904**
1	3	2	2	0	4	0.2752
1	1	2	2	2	5	0.2041
1	1	2	2	2	6	0.2041
1	1	2	4	0	7	0.2583
1	1	2	2	2	8	0.2041
8	14	18	16	8		**C = 2.0155**

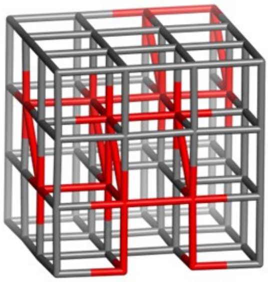
$H(G_2) = 7x + 9x^2 + 8x^3 + 4x^4$

$H'(G_2) = 7+18+24+16 = 65 = W(G_2)$

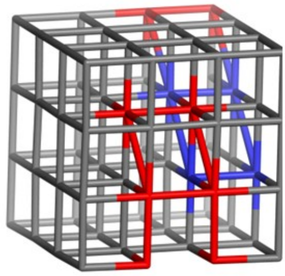
Figure 2. LC matrix for the graphs G_1 and G_2 and their Hosoya polynomials.

Results

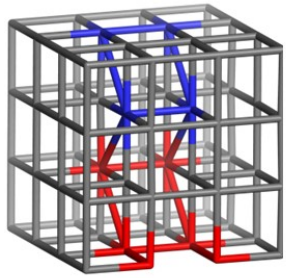
We suppose here a student is learning in three libraries M1, M2 and M3 (Figure 3); they have the same number of shelfs/edges and grids/vertices but different stears between them. Which one is the best organized/built so that the walk length for inspecting the books is minimal?



M_1; C=7.9609; W=7352



M_2; C=7.9636; W=7348



M_3; C=7.9296; W=7246

Figure 3. Graphs of library buildings: v=64; e=146; $C(M_2)>C(M_1)>C(M_3)$

The answer is: that library showing the minimal C-value. In our case, the non-increasing ordering of centrality is: $C(M_2) > C(M_1) > C(M_3)$. This is in agreement with the ordering given by the centrality criteria of Bonchev.

The ordering provided by the Wiener index W (i.e. the sum of all distances in G) is different: W(M1)>W(M2)>W(M3), with the meaning the least W-value correspond to the least sized structure, but is diferent from the centrality ordering. Clearly, the two orsering are not „reciprocal" to each other, as could be suggested by the values in Figure 2.

Computations have been done by the TOPOCLUJ software package. (Ursu, Diudea 2005)

Conclusions

Optimizing the learning activity, by minimization of the walk length for getting the books, as an important fact in the education process, can be achieved by using the Graph Theory. The distance matrix provides a distance-based parameter, the Wiener index (i.e. the sum of all distances in the graph). This index is an indicator of the size of a given structure/graph. A better answer was given by the centrality index C, that provides an ordering of walk length more related to the centrality criteria based on the distances in graph. It is necessary to imagine more cases, with various access ways, to complete the picture of optimized learning process, as a function of the time expense for books inspection.

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Notes on the authors

MIRANDA P. VLAD is an Associate Professor PhD at the Faculty of Economic Sciences Cluj-Napoca, “Dimitrie Cantemir” Christian University Bucharest, Romania. She holds a PhD thesis in Cybernetics and Economic Statistics. She published numerous articles and studies in journals from Romania and abroad. She is a member in „Asociatia pentru Promovarea Învatamântului de Informatica Economica - INFOREC - in the „Asociația Generală a Inginerilor din România”AGIR and in the „European Society of Mathematical Chemistry”.
E-mail: mirandapv@yahoo.com

MIRCEA V. DIUDEA is a PhD Professor at the Faculty of Chemistry and Chemical Engineering, Babes-Bolyai University, Cluj-Napoca, Romania. He is the author of a large number of books, articles and studies in journals from Romania and abroad. In 1986 he established the TOPO GROUP CLUJ, and in 2007 founded the European Society of Mathematical Chemistry ESMC, of which first president is.
E-mail: diudea@gmail.com