DETERMINATION OF THE OPTIMUM PRODUCTIONS SURFACE (MONOFACTORIAL MODEL) HAVING AN INITIAL FIXED CAPITAL

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Abstract: *Optimum size of the area destinated for agricultural crops is a actual subject in sustainable rural development strategies; most often presents interest to estimate a priori the areas used starting from an fixed capital, situations commonly encountered in allocation of funds practicing destinated to rural development projects. Mathematically, optimization models can have multiple starting points; here the optimization study is realized starting from the condition of using the fixed capital in amount S, originally established, strictly, for the entire production process, in case in witch there is no question of the existence of a minimum value of production. The proposed model is based on the restriction caused by the equality between initial fixed amount and product between optimal surface and the amount of fixed and variable costs expressed per unit area. It will be use a single factor production function, as a function of the second degree, which will express the dependence between production per unit area dependent by the variable expenses for fertilization soil. The original data are those from the 2013 agricultural year level.*

Keywords: *rural development, optimum surfaces, agricultural crops, monofactorial model*

INTRODUCTION

When we speak about optimizing agricultural production surfaces problem, the problems can be totally different as design: direct observations on size of the size of some other similar farms and correlation of their dimension with economic benefits, optimize the surface in the concrete situation of the existence of a stabile contract on the selling of a given quantity from the final product, etc. There are also the case in witch is not required a minimum quantity of production, the restriction being given by an initial investment strictly fixed. So the

product between quantity economic optimum related of nitrogen fertilization respective N_{optim}, p_N the price of fertilization unit, to witch are add the fixed amounts per unit area (s_f) with optimum production surface will be equal to S, an amount initially fixed (capital) for production: $N_{optim} \cdot p_N \cdot s_{optim} + s_f \cdot s_{optim} = S$. The object of this paper is to determine the optimal area destinated to an agricultural production, starting from the fixed capital S, for the current, concrete situation. The model has on base previous theoretical studies, here production function having a monofactorial expression, $Q = aN^2 + bN + c$ (a simplified case in witch we consider that production is directly and significantly influenced by a single factor) (Rujescu C., et al., 2005.2014)

Optimization problems are not a novelty; they have an appreciable length and practically we can say that its foundations were laid together with the development of differential calculus concepts and theory foundation regarding the determination of extreme points of real functions by one or more real variables (Floudas 2009, Intriligator 2002, Steven 2008). Their use is found permanent in current issues from agriculture, field that shows a continuous economic instability (Otiman, 2005). Thus, not infrequently a farm manager, before performing a calculation on contracting an amount for investments in agriculture (such as projects funded with EU founds) is faced with a real problem, namely the use of a sum date (strictly) originally fixed, in an optimal way.

MATHEMATICAL METHOD

To calculate the maximum profit that can be achieved in a farm, in the theoretical model case we don't considered the exploitation price of the products obtained as being constant (Rujescu C., 2005); we started from the hypothesis that the price depends on market demand, which is the lower as higher is the price and vice versa. Usually this dependence can be expressed mathematically by the equation: p = p (k), where "p" represents the price, and "k", the quantity existent on the market.

With the notations above, the dependence of the profit it can also be expressed by relationship:

$$Pr = (aN^2 + bN + c)p(k) - p_N N - s_f,\ a < 0 \quad (1)$$

Canceling the partial derivate with respect to N, of the profit function we obtain:

$$\frac{\partial Pr}{\partial N} = \frac{\partial\left[\left(aN^2 + bN + c\right)p(k) - p_N N - s_f\right]}{\partial N}$$

$$\frac{\partial Pr}{\partial N} = \left(2aN + b\right)p(k) - p_N = 0$$

,

So a linear relationship minus the expression of the optimal value:

$$2aNp(k) + bp(k) - p_N = 0 \Rightarrow 2aNp(k) = p_N - bp(k)$$

So

$$N_{optim} = \frac{p_N - bp(k)}{2ap(k)}$$

.

So the maximum profit per unit area will be determined by substituting this optimum value of N in relation (1):

$$Pr_{max/\,ha} = \left(aN_{optim}{}^2 + bN_{optim} + c\right)p(k) - p_N N_{optim} - s_f,\ a < 0$$

So we have:

$$Pr_{maxi\,ha} = a\left(\frac{p_N - bp(k)}{2ap(k)}\right)^2 p(k) + b\left(\frac{p_N - bp(k)}{2ap(k)}\right)p(k) + cp(k) - \frac{p_N{}^2 - bp_N p(k)}{2ap(k)} - s_f$$

In agreement with the initial condition, given by the case in witch the manufacturer has a fixed amount "S" for allocation the production process and in the case we consider that it has sufficient land area (optimal area for culture „s_{optim}" to be less than the total available s_T), we have:

$$N_{optim} \cdot p_N \cdot s_{optim} + s_f \cdot s_{optim} = S$$

,

Meaning

$$\frac{p_N - bp(k)}{2ap(k)} \cdot p_N \cdot s_{optim} + s_f s_{optim} = S$$

from where optimal surface expression is derived:

$$s_{optim} = S\left(\frac{p_N - bp(k)}{2ap(k)} \cdot p_N + s_f\right)^{-1} \quad (2)$$

or

$$s_{optim} = \frac{2aSp(k)}{p_N(p_N - bp(k)) + 2ap(k)s_f} \quad (2')$$

So profit that is obtained if is cultivated this surface will be given by the expression:

$$Pr_{max} = \left(p(k)\left(a\left(\frac{p_N - bp(k)}{2ap(k)} \right)^2 + b\left(\frac{p_N - bp(k)}{2ap(k)} \right) + c \right) - p_N\left(\frac{p_N - bp(k)}{2ap(k)} \right) - s_f \right) \cdot \frac{2ap(k)S}{p_N(p_N - bp(k)) + 2ap(k)s_f}$$

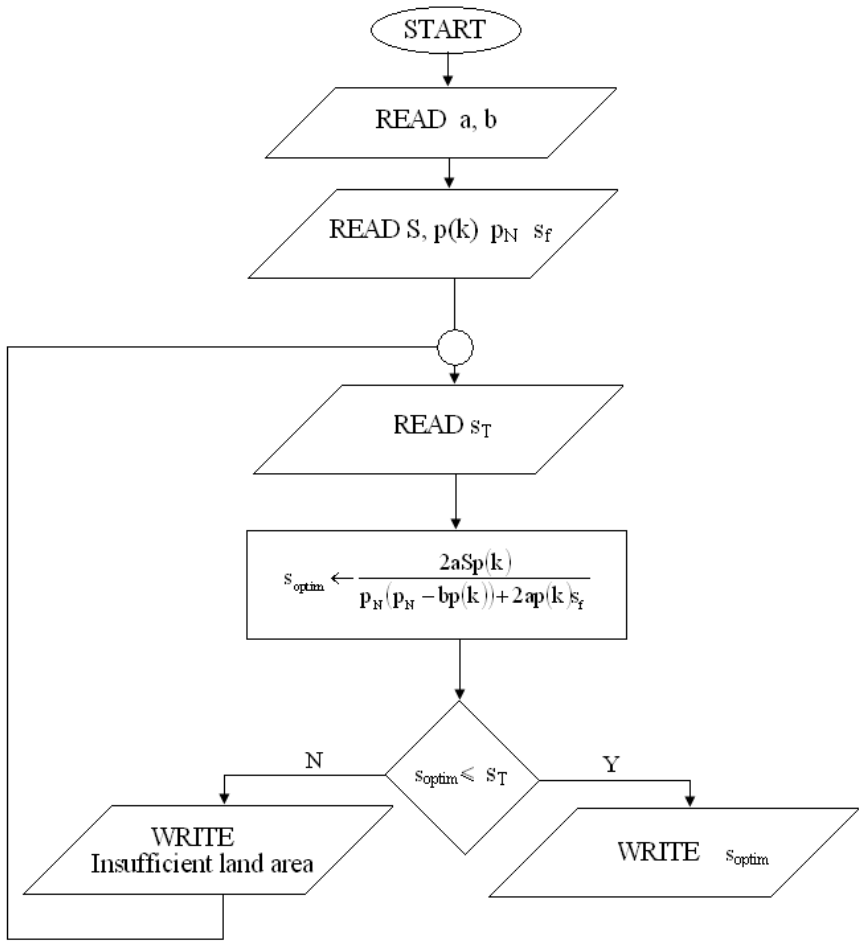

Figure 1 Flow chart corresponding to mathematical model

RESULTS AND DISCUSSION

We will study an example, for a function that indicates the influence of nitrogen doses on wheat production, on a cambic chernozem soil in kg/ha (Borlan, Hera, 1982, 1984) having the expression (fig. 2):

$$y = -0.073\ N^2 + 13.57\ N + 4191.$$

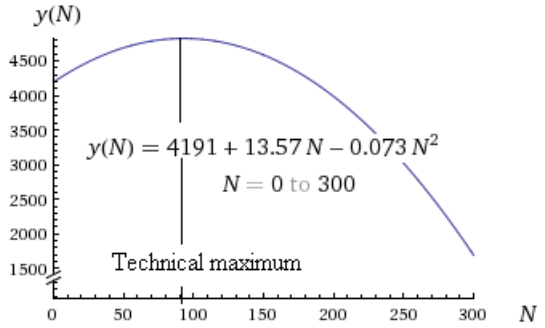

Figure 2 The influence of nitrogen quantity on the production of wheat (kg/ha)

Data from 2013 level indicates the price of ammonium nitrate fertilizers 33.4% approximated to 331.8 euro/tone and fixed costs per hectare of wheat of 270 euro. Suppose set the price of wheat, p (k) = 0.3 euro/kg (inss.ro, anaf.ro, apia.org.ro).

Substituting data in relation 2 is obtained:

$$s_{optim} = \frac{2a \cdot Sp(k)}{p_N(p_N - bp(k)) + 2a \cdot p(k)s_f} = \frac{2(-0,073) \cdot S \cdot 0,3}{0,3318 \cdot (0,3318 - 13,57 \cdot 0,3) + 2(-0,073) \cdot 0,3 \cdot 270}$$

We obtain the data shown below.

S available (euro)	Optimal surface (ha)
10000	33,52
12000	40,22
14000	46,92
16000	53,63
18000	60,33
20000	67,04
25000	83,80

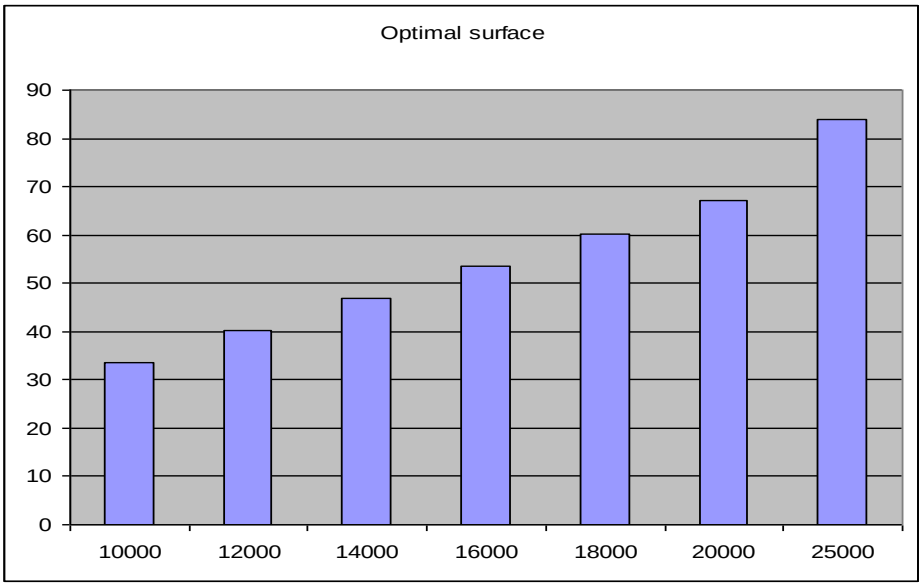

Figure 3 Optimal surface vs available capital.
Results obtained after values of available capital

CONCLUSIONS

The calculations of the model are easy algorithm-based, they also involving a smooth transposition without difficulties in a software application. The flowchart shown in Figure 1 indicates in fact a test of the classification of optimal area in total area and generating the value of optimal surface in the concrete case.

Current bibliography of the field indicates many monofactorial production functions, which expresses the dependence between production and a major influential factor that can be used in studies of this type.

Also, these calculations can be useful at farm level, where economic context require that necessity of some previous optimization calculations. One of the situations encountered in practice, which can be implemented in such a model, represent the calculations of projects with EU funds, when the manager will face the situation of optimal using of a fixed amount, initially set. Just this aspect justifies equality (strict) from the initial restriction: "the equality between fixed capital and product between optimal surface and the total costs per unit area".

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